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Estimation of Intravascular Blood Pressure Gradient by Mathematical Analysis of Arterial Casts

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INTRODUCTION

In a previous report from the Institute Furuyama[2] described a histometrical method with which the strength of arterial muscular coat could be estimated with an accuracy sufficient to define medial hypertrophy in arterial hypertension. On the basis of the obtained results increased medial strength of arteries was regarded to be an anatomical estimate of elevated blood pressure in arterial systems. However, an exact correlation of medial strength to intravascular blood pressure was not possible, because there was no exact investigation available for the purpose. In the present report, an attempt was made theoretically to estimate intravascular blood pressure by means of mathematical treatments of arterial casts formed by acrylic resin infusion.

In analytical studies of arterial wall, the most fundamental relation is given by Laplace's equation, $T=PR$, where T is the tension exerted on arterial wall, P is intravascular blood pressure and R is arterial radius. Accordingly, the structure of arterial wall is to be investigated in reference to the two physical factors P and R. Direct measurements of arterial blood pressure are practically of very limited application except on large arterial branches, and the aspect of intravascular blood pressure gradient is at present only accessible to theoretical treatments. Because the physical conditions of the arterial system characterized by pulsatile flow in tubes with elastic walls do not strictly satisfy the requirements for an application of Hagen-Poiseuille's formula, which has been generally employed for the estimation of intravascular blood pressure gradient, the results derived from it are sometimes regarded to lack reliability. The deviations from the requirements are probably most pronounced in large arterial branches, where turbulent blood flow may develop at the maximum velocity of blood stream in the

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systolic phase. However, as Reynolds number is rapidly lowered and the amplitude of pulsation is reduced with progressive arterial divisions, we can assume that blood flow is continuously laminar, except in the region where blood pressure drop is only insignificant, whether blood flow is laminar or turbulent. In the present investigation of intravascular blood pressure gradient, there is little reason to discard the formula, so far as the above deviations in physical conditions are concerned.

The main inconveniences in the practical application of the formula consist rather in the following two points. One of them concerns the determination of arterial radius. In the classical Hagen-Poiseuille's formula blood pressure difference is inversely proportional to the fourth power of arterial radius. Because of the high exponent, small errors in the determination of arterial radius cause surprisingly large deviations in the result. In spite of the strict requirement, our means of direct measurements of arterial radius in living organisms are very restricted and can hardly give informations adequate for our purpose. In autopsy specimens, arterial radius is remarkably reduced on account of irregular post-mortem arterial constriction, and there is no method sufficiently reliable to reduce arteries in autopsy cases to their living states. In the present investigation, arterial radius is estimated by direct measurements on arterial cast of acrylic resin. As the resin must be infused under a pressure which sometimes surpasses physiological blood pressure, arteries are more or less distended, and it is evident that the radius of the cast does not correspond to that of living artery. However, as will be discussed later in the last section of this report, the discrepancy between casts and living arteries can be corrected to a considerable extent by theoretical treatments of blood pressure drop. For the time being, the results of direct measurements on arterial cast are preliminarily employed as the estimate of arterial radius.

The other difficulty is that the classical Hagen-Poiseuille's formula is only applicable on an unbranching tube of uniform radius. In the study of actual arterial systems with rapidly progressive ramification, the formula cannot be introduced without an adequate information about the blood flow of individual branches. In previous works blood flow of individual arterial branches was estimated by the following procedure. The mean radius of anatomically defined sites of an arterial system was determined by actual measurements. On the other hand, the number of arterial branches of the corresponding anatomical region was counted to give the sum of arterial cross section area. From the obtained results the mean blood flow of individual arterial branches in each of the selected orders of arterial ramification was determined, if the total blood flow of the arterial system and the radius of the arterial trunk were given. In this way, calculation of intravascular blood pressure was possible from the arterial trunk to capillaries. Even in more recent works of this direction, the principle employed

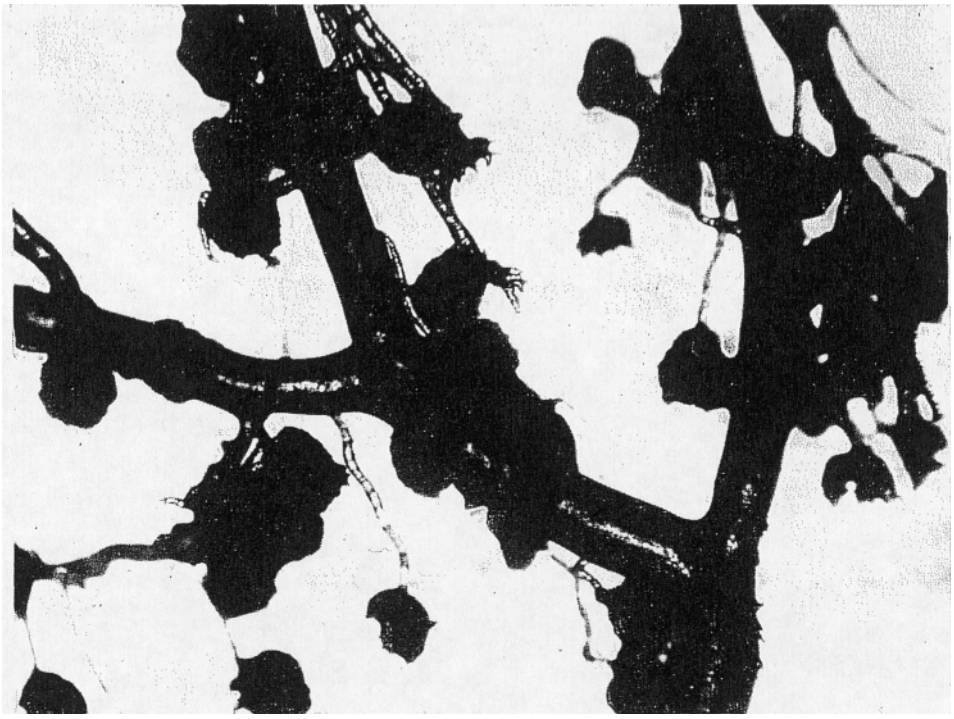

Fig. 1. The aspect of branching of interlobular arteries of the kidney. The entire region in the figure belongs to interlobular artery in anatomical sense of the word. Note successive reduction of the radius and the difficulty in defining the "mean radius" of interlobular arteries without highly artificial selection of branch group.

remained essentially the same as in the works in the early decades of this century. The major difficulty attaching to the method is that the determination of the mean radius is hardly possible without a highly arbitrary and artificial selection of arterial branch groups. If we observe actual arterial ramification, it is easily comprehended that the radii of arterial branches at a certain anatomically defined site, for example those of interlobular arteries of the kidney, are represented by a continuous and uninterrupted series of values from the preceding arterial branch group to that of the next order. There is no natural border in arterial radius to separate anatomically defined arterial groups. In such a condition, it is hardly appropriate to employ the "mean radius" of a certain branch group in a theoretical treatment of blood flow, because of the uncontrollable errors arising from the assumption. Although the investigations hitherto reported revealed an approximate pattern of intravascular blood pressure gradient characterized by abrupt fall of blood pressure in the arteriolar region, comparative studies of various arterial systems were almost impossible because of the inaccuracy in the determination of factors in Hagen-Poiseuille's formula. In the present report, an attempt was made to facilitate the application of the formula to a system of branching tubes by expressing blood flow as a power function of arterial radius. This procedure warranted more exact mathematical treatments of arterial

systems than the method in previous works and made possible to characterize the pattern of intravascular blood pressure gradient of different arterial systems.

BLOOD FLOW AS A FUNCTION OF ARTERIAL RADIUS

The blood supply of organs is accomplished by branching arterial systems and it is expected that the radius of an arterial branch has some bearing on the quantity of the tissue receiving blood from it. Because the mean blood flow for equal tissue quantities of an organ can be assumed to be equal, the mean blood flow of an arterial branch must be in some way correlated to its radius. The relation was confirmed in the kidney by the following procedure. By means of serial histological sections of a normal human kidney, the radius of interlobular arteries of various size was determined on their exact cross sections together with the number of glomeruli belonging to them. As arteries in autopsy specimens were in a state of post-mortem constriction, the length of waving internal elastic membrane was determined by attaching thin cotton thread on the magnified depictions of arterial cross sections, divided by 2π and used as the estimate of arterial radius. When we took the abscissa for arterial radius and the ordinate for the number of glomeruli and plotted the results of measurement on a logarithmic scale, distinct linear regression was confirmed as in Fig. 2. On the assumption that the mean blood flow of all glomeruli was equal, the number of glomeruli belonging to an arterial branch could be regarded to be proportional to the mean blood flow of the branch. Accordingly, the relation of the radius of an arterial branch to its mean blood flow was expressed by a general formula $Q=qr^n$,

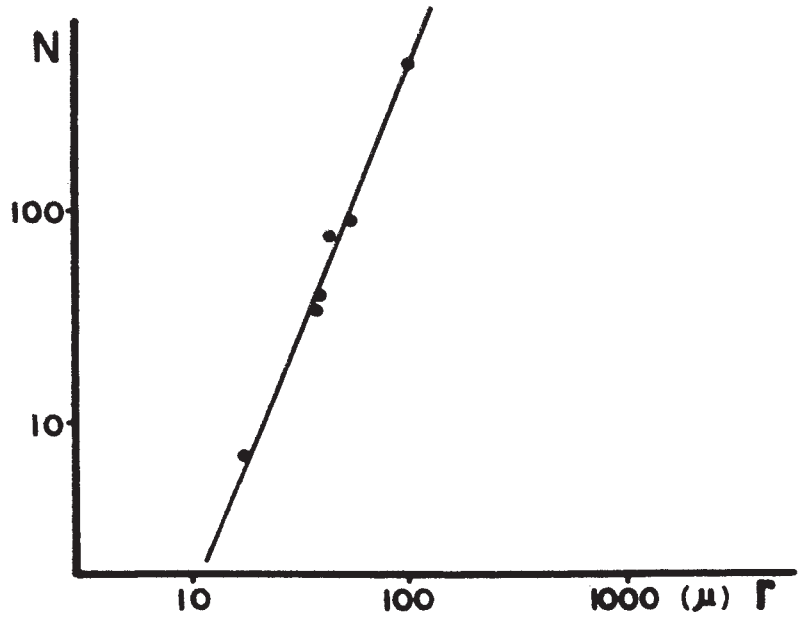

Fig. 2. The relation between the radius r of interlobular arteries and the number of glomeruli N belonging to each artery is presented on a logarithmic scale.

in which Q was the mean blood flow of a given arterial branch, r was the radius and q and n were constants. In the case of Fig. 2, the value of n was approximately 2.4. The meaning of q will be discussed later. It was thus demonstrated, that the mean blood flow of an arterial branch could be expressed by a power function of its radius. However, this method was not only extremely time consuming and not adequate for an exact determination of $\bar{n}$ (mean n) on account of limited number of observations, it was not applicable on other organs than on the kidney, because they were devoid of anatomical structures such as glomeruli where the uniformity of the mean blood flow could be postulated. A different approach to the problem was attempted as follows. When an arterial branch with the radius r and the mean blood flow Q is divided into a number of subbranches with radii $r_1, r_2, \ldots, r_m$ and with the mean blood flow $Q_1, Q_2, \ldots, Q_m$, respectively, we obtain $Q=Q_1+Q_2+\ldots.+Q_m$. If we assume that the mean blood flow is proportional to the n-th power of radius, then it is determined by $Q=qr^n$, $Q_1=qr_1{}^n$, $Q_2=qr_2{}^n, \ldots, Q_1=qr_1{}^n$. Accordingly, we obtain:

$$r^n = r_1{}^n + r_2{}^n + \ldots\ldots + r_m{}^n. \qquad (1)$$

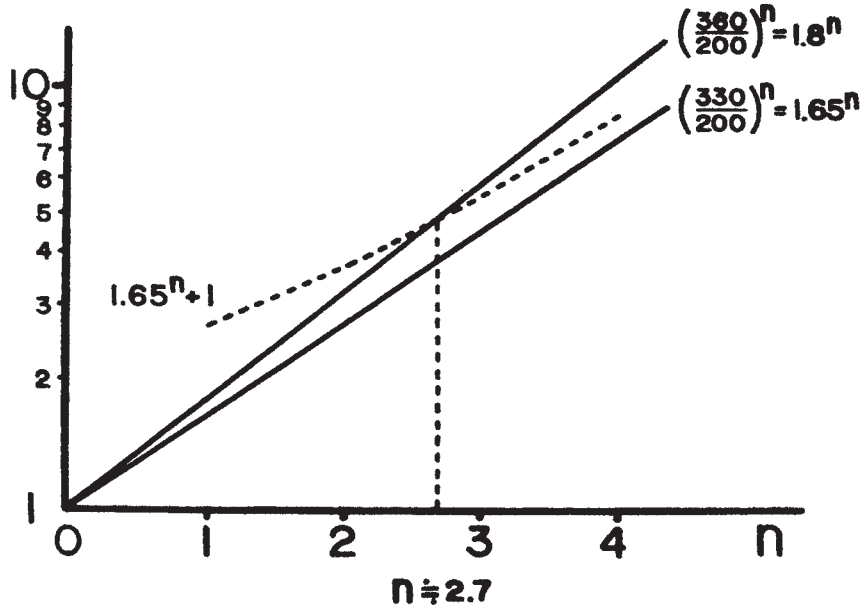

Fig. 3. Approximate value of n is to be determined by graphical analysis on a semi-logarithmic scale. Suppose an arterial branch of 360μ in radius which divides into two subbranches of 330μ and 200μ in radius. Divide the radii of two larger branches with the radius of the smallest branch, and we obtain 1.80 and 1.65. Take Cartesian scale for n and logarithmic scale for the n-th power of the above ratios and draw two straight lines corresponding to 1.80^n and 1.65^n. Plot the points of 1.65^n+1 on the diagram by giving several different values for n. The series of the points make a curve which passes 2 of the logarithmic scale at $n=0$ and asymptotically approaches to 1.65^n with increasing n. Determine the crossing point of the curve with the line of 1.80^n, and we obtain 2.7 as an approximate solution of $360^n=330^n+200^n$. The application of the method can be extended to divisions of more than two subbranches.

In other words, on the assumption that the mean blood flow of an arterial branch is proportional to the n-th power of its radius, the n-th power of its radius must be equal to the sum of the n-th power of the radii of its subbranches. If we assume that the mean blood flow of arterial branches of equal radii is the same within a certain organ, the relation $Q=qr^n$ is easily deduced from (1).

The equation (1) to determine the value of n when r, r_1, r_2, . . . , r_m are given, is not to solve except in special cases, but approximate values on n with desired accuracy can be obtained with an electron computer. When an electron computer is not available, the values can be estimated by the graphical analysis on a semi-logarithmic scale. The method is demonstrated in Fig. 3. It was further very practical to draw a nomogram like that in Fig. 4, with which a large number of measurements on arterial casts could be treated without difficulty.

In Fig. 5, the values of n obtained from the measurements on the superior mesenteric artery of the autopsy case of a 33-year-old non-hypertensive male are demonstrated. The distribution of n can be regarded to be a distribution of errors around a certain value, but it is at the same time distinctly asymmetric. The asymmetry is due to rapid elevation of n in the region of its larger values with only slight increase in the radii of subbranches. If we take an arterial

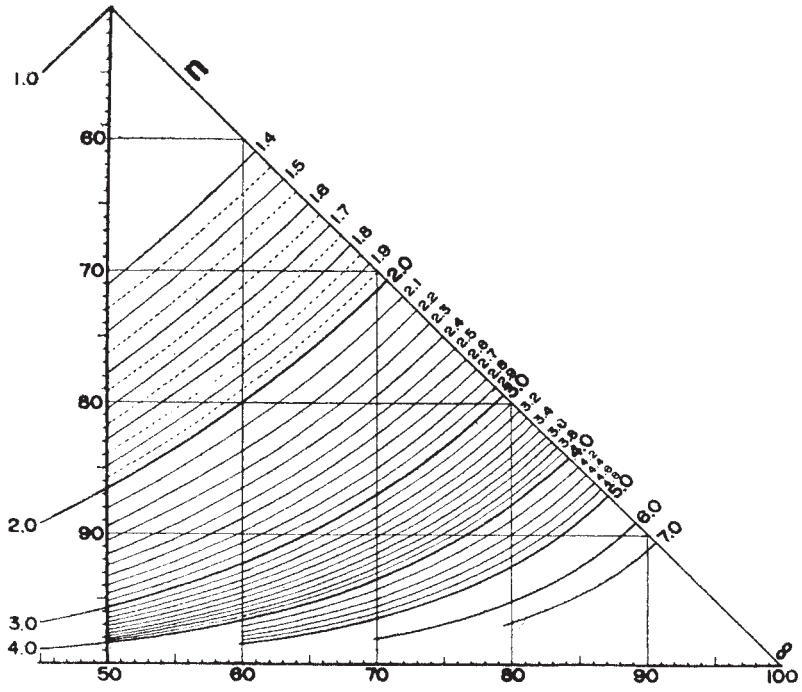

Fig. 4. The nomogram for the determination of n. Suppose an arterial branch of r in radius which divides into two subbranches of r_1 and r_2 ($r_1 \geq r_2$) in radius. Transform the equation $r^n=r_1^n+r_2^n$ to $1=(r_1/r)^n+(r_2/r)^n$. Take $100\times r_1/r$ on the vertical axis and $100\times r_2/r$ on the horizontal axis. Draw a straight line vertical to each axis through the points, respectively, and determine the crossing point of the two straight lines, which gives the approximate value of n on the nomogram. (Planned and drawn by Fukasawa.)

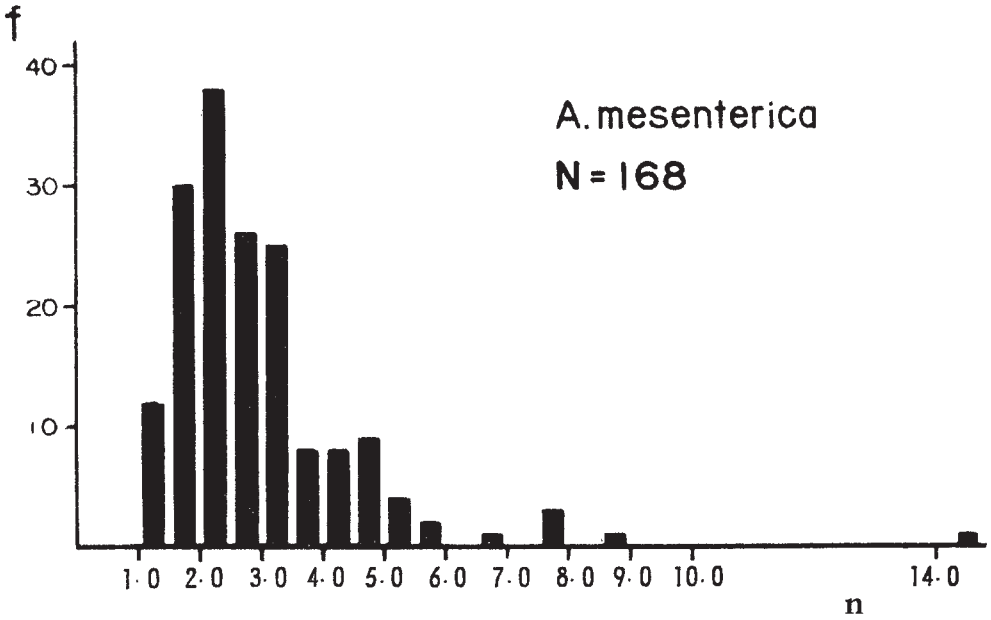

Fig. 5. The frequency distribution of n of a normal mesenteric artery. Note distinct asymmetry of the distribution.

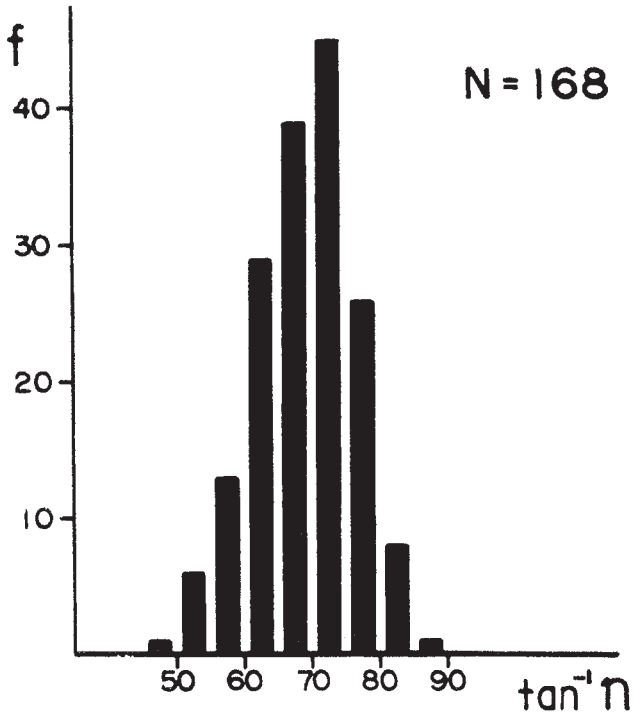

Fig. 6. The distribution of n of the same sample group as in Fig. 5 after arctangent transformation. The distribution is almost perfectly symmetrized.

branch with radius r, which divides into two subbranches of identical radius r_i, the formula (1) will be $r^n=2r_i{}^n$. Fixing r to a constant value and giving to r_i successively increasing values from $r/2$ to r, we obtain a series of corresponding n ranging from 1 to infinity. Now we set the values of $r_1, r_2, \ldots, r_i$ so, that the interval of

Table I–a.

	Autopsy number	Age and sex	Organ	Number of samples N	$\bar{n}$	$\bar{x} = \arctan \bar{n}$ (radian)	Variance $Sx^2/(N-1)$
$r \geqq 100\mu$	202/61	16 ♂	Kidney	70	**2.75**	1.2217	0.004910
	34/62	33 ♂	Kidney	105	**2.74**	1.2206	0.004847
	136/62	29 ♂	Kidney	140	**2.56**	1.1978	0.006508
	200/62	27 ♂	Kidney	105	**2.74**	1.2210	0.007113
	136/62	29 ♂	Intestine	193	**2.63**	1.2077	0.008270
	271/62	21 ♂	Intestine	105	**2.57**	1.2002	0.005910
	200/62	27 ♂	Intestine	105	**2.86**	1.2339	0.006155
	136/62	29 ♂	Muscle	159	**2.69**	1.2152	0.006196
	271/62	21 ♂	Muscle	105	**2.86**	1.2339	0.004499
	297/61	54 ♀	Cerebral Cortex	106	**2.67**	1.2130	0.008110
	297/61	54 ♀	Basal Ganglion	61	**2.64**	1.2091	0.004822
	136/62	29 ♂	Pancreas	55	**2.54**	1.1961	0.012000
	271/62	21 ♂	Heart	116	**2.51**	1.1913	0.007021
	102/62	42 ♂	Lung	108	**2.66**	1.2109	0.005221

Analysis of variance

Source of variation	Degrees of freedom	Sum of squares	Variance	F
Between	13	0.1587	0.01221	1.87*
Within	1519	9.9398	0.00654	
Total	1532	10.0985		

	Autopsy number	Age and sex	Organ	Number of samples N	$\bar{n}$	$\bar{x} = \arctan \bar{n}$ (radian)	Variance $Sx^2/(N-1)$
$r < 100\mu$	202/61	16 ♂	Kidney	73	**2.53**	1.1951	0.012803
	34/62	33 ♂	Kidney	86	**2.86**	1.2340	0.011458
	136/62	29 ♂	Kidney	83	**2.74**	1.2206	0.012322
	200/62	27 ♂	Kidney	105	**2.70**	1.2126	0.011227
	136/62	29 ♂	Intestine	140	**2.68**	1.2131	0.006965
	271/62	21 ♂	Intestine	105	**2.68**	1.2143	0.004339
	200/62	27 ♂	Intestine	105	**2.87**	1.2259	0.006442
	136/62	29 ♂	Muscle	105	**2.83**	1.2306	0.006908
	271/62	21 ♂	Muscle	105	**2.87**	1.2360	0.005051
	297/61	54 ♀	Cerebral Cortex	106	**2.79**	1.2268	0.008121
	297/61	54 ♀	Basal Ganglion	147	**2.61**	1.2045	0.008559
	136/62	29 ♂	Pancreas	66	**2.73**	1.2200	0.012417
	271/62	21 ♂	Heart	108	**2.82**	1.2303	0.009440
	102/62	42 ♂	Lung	121	**2.47**	1.1867	0.008987

Analysis of variance

Source of variation	Degrees of freedom	Sum of squares	Variance	F
Between	13	0.2954	0.02272	2.65*
Within	1441	12.4048	0.00861	
Total	1454	12.7002		

TABLE I-b.

Class	Number of samples N	$\bar{n}$ of class	Confidence interval for $\bar{n}$ at 95% level	$\bar{x}$ of class	Variance
$r \geqq 100\mu$	1533	**2.66**	2.62—2.69	1.2107	0.006592
$r < 100\mu$	1455	**2.71**	2.67—2.75	1.2175	0.008735

Analysis of variance

Source of variation	Degrees of freedom	Sum of squares	Variance	F
Between	1	0.0350	0.0350	4.58*
Within	2986	22.8337	0.0076	

$r_i - r_{i-1}$ is uniform throughout the series, or that $r_1, r_2, \ldots, r_i$ make an arithmetical series. The corresponding intervals of n, $n_i - n_{i-1}$, increase rapidly, if r_i approaches to r. Accordingly, in order to treat the values of n statistically and to correlate the deviation of n to the errors of arterial radius, some transformation is required to make the interval of $n_i - n_{i-1}$ nearly uniform. Arctangent transformation was found to be the most effective for this purpose, and it could be confirmed that $\tan^{-1} n_1$, $\tan^{-1} n_2, \ldots\ldots, \tan^{-1} n_i$ approximately made an arithmetical series. The distribution of $\tan^{-1} n$ or X of the same sample group as in Fig. 5 is demonstrated in Fig. 6. Asymmetry of the distribution of n is almost entirely eliminated and it is now possible to treat the values of X as forming a normal distribution around the mean $\bar{x}$, to apply necessary statistical treatments and to determine $\bar{n}$ corresponding to $\bar{x}$ as the mean of n.

In Table I, the values of $\bar{n}$ of several normal arterial systems are presented. It is demonstrated that the values of $\bar{n}$ are nearly the same in all examined arterial systems without any appreciable difference according to organs. Analysis of variance reveals, however, that between class deviations of $\bar{n}$ are too large to be attributed to within class deviations of n. This indicates that the value of $\bar{n}$ may be different according to examined arterial samples. It is most probably due to some differences in the condition under which resin is infused. The values of $\bar{n}$ further exhibit some difference according to regions in reference to arterial radius, and the region $r < 100\mu$ has slightly, but statistically significantly, higher values of $\bar{n}$. An elevation of $\bar{n}$ of this range in the region $r < 100\mu$ does not cause a remarkable difference in the final results of blood pressure drop, which will be later calculated by our method. Accordingly, the difference between the two regions is neglected and the value of $\bar{n}$ is fixed to 2.7 throughout the whole range of arteries and throughout all the examined organs or arterial systems.

In the determination of n, we experienced that in some instances a branch did not reduce its radius even after a subbranch is divided. The majority of such cases was caused by divisions of too small subbranches. The minimum radius of

a subbranch which will perceptibly influence the radius of the original branch can be calculated as follows. If an arterial branch of r in radius divides into two unequal subbranches of r_1 and r_2, respectively, the relation among them is determined by $r^{2.7}=r_1^{2.7}+r_2^{2.7}$, if n is 2.7. We assume that r_2 is smaller than r_1 and is equal to αr. Our purpose is to determine the minimum value of α in order that r_1 is perceptibly smaller than r. If we regard the minimum perceptible reduction of radius to be 5%, the value of α can be determined from $\alpha^{2.7}>1-(0.95)^{2.7}$. The result, $\alpha>0.47$, indicates that a subbranch must have a radius larger than 47% of the original branch, in order that the reduction of the radius of the original branch is confirmed. At the division of a subbranch smaller than that, the determination of n has little meaning, because it is perfectly obscured by the error in the determination of arterial radius. In the present investigation, arterial divisions with subbranches smaller than 50% of the original branch in radius were not included in the determination of n. In the remaining small number of cases, reduction of arterial radius was not noticed even after the division of a large subbranch over 50% of the original branch in radius. In such cases n was determined including the next subdivision where reduction of radius was noticed. The procedure is demonstrated in Fig. 7.

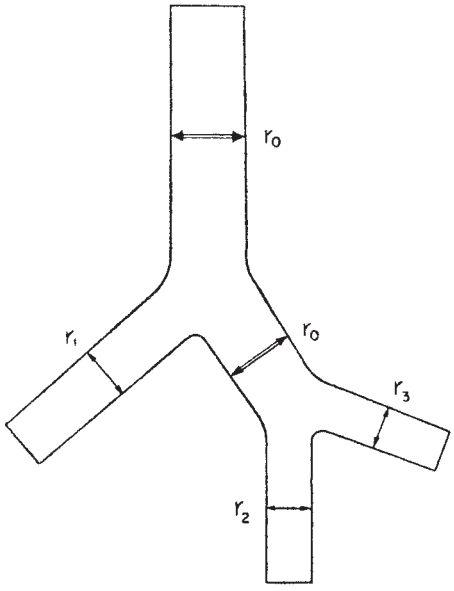

Fig. 7. When an arterial branch of r_0 in radius does not perceptibly reduce its radius even after the division of a subbranch of r_1 in radius ($r_1>r/2$), the value of n at this division is determined including the next division where r_0 diminishes its value to r_2 and r_3. The equation to solve is $r_0^n=r_1^n+r_2^n+r_3^n$, instead of $r_0^n=r_0^n+r_1^n$.

As only $\bar{n}$ is used in the subsequent part of the report, $\bar{n}$ is written as n for the sake of simplicity.

From the relation $Q=qr^n$, the number of arterial branches of r in radius is given by $Q_t/Q=(r_t/r)^n$, Q_t and r_t being the blood flow of the organ and the radius of arterial trunk and Q and r those of the branches, respectively. The sum of cross section area A of arterial lumen at the radius r is determined by $A=\pi r^2 \cdot (Q_t/Q)=\pi r_t^n \cdot r^{2-n}$.

With the determination of n, it is now possible to calculate the value of q in $Q=qr^n$ by giving the value of blood flow of an organ to Q and the radius of the trunk of organ artery to r. The radius of arterial trunk can naturally be determined on the arterial cast. However, as a catheter must be inserted into the trunk for resin infusion, the estimation of the radius is often difficult on account of the deformity due to the manipulation. Besides, acrylic resin of very high viscosity must be used for perfect formation of arterial cast of the trunk, so that simultaneous infusion of small arteries is seriously affected. In the present investigation, the value was substituted by the result of the following histometrical determination. Histological slides of exact cross sections of arterial trunks were magnified, projected and depicted. Internal elastic membrane was accurately delineated and measured by attaching thin cotton thread on the depictions. The obtained values were divided by 2π and reduced to the original dimension in the slides by calculation. Because paraffin sections were used for the determination, the obtained values were adjusted by a coefficient to remove them of the effect of shrinkage which accompanied the preparation of histological slides. The value of the coefficient was estimated to be 1.15, on the basis of our own investigations with arterial trunks. As two different principles are introduced for the determination of arterial radius, a comparative study is required about their results. The problem will be discussed later in the last section of this report systematically. In this place it is only pointed out, that the two methods give nearly identical results, so far as arterial trunks are concerned.

In Table II, the mean values for the radius of arterial trunks are presented together with their confidence intervals. The values for cerebral arteries are given for a hypothetical arterial trunk which will be made by the union of two internal carotid and two vertebral arteries and can be calculated by $r_t^{2.7}=\Sigma r^{2.7}$, in which r represents the radius of each of the four main cerebral arteries. In the heart r_t was determined with the three main branches of coronary artery in the same way. For superior mesenteric and coeliac arteries the blood flow was calculated from the blood flow of the liver by assuming that it is proportional to $r_t^{2.7}$ of each artery. When arterial radius is expressed in μ, and Q in ml/sec., q in $Q=qr^{2.7}$ represents blood flow in ml/sec. of an arterial branch of 1μ in radius. In actual arterial systems there are of course no such small branches, but we can nevertheless define the blood flow of these hypothetical small arteries as specific

TABLE II.

Organ or artery	Number of samples	Mean radius r_t (μ)	Blood flow ml/sec.	q 10^{-9} ml/sec.
Brain	8	2872±162	14.0	6.4 (5.6—7.5)
Heart	2	1873	3.8	5.5
Kidney	23	2726±131	8.3	4.4 (4.1—5.0)
A. mesenterica superior	16	3061±205	10.8	4.2 (3.5—5.0)
A. coeliaca	17	3235±160	12.5	4.2 (3.7—4.8)

In the last column of the table, confidence intervals of q in reference to the errors of r_t are given in parentheses.

blood flow and use the values in our calculation of blood pressure gradient. In Table II, the values of q are given calculated from $Q=qr_t^{2.7}$, together with their confidence intervals in reference to possible errors in the determination of the mean arterial radius. The result means that, strictly speaking, q is an organ specific constant, but it is rather surprising that the values of q of the examined organs are nearly equal in spite of different anatomical aspects of arteries according to organs. This indicates, that arterial branches of equal radii have practically equal mean blood flow, irrespective of organ difference.

TRANSFORMATION OF HAGEN-POISEUILLE'S FORMULA

In advance of the transformation of Hagen-Poiseuille's formula, a brief notice is necessary on the so-called "anomalous viscosity" of blood. The blood is not a homogeneous fluid but contains erythrocytes. It was demonstrated by Dix and Scott Blair[1], that for fluid containing corpuscular element the resistance to the flow in narrow tubes was lower than was required by Hagen-Poiseuille's formula. Haynes[3] applied a correction to viscosity coefficient of the blood and observed satisfactory agreement of the theoretical calculation to experimental observations. The correction is:

$$\eta' = \eta \Big/ \left(1 + \frac{r'}{r}\right)^2,$$

in which η' is the apparent viscosity coefficient in a narrow tube, η is the real viscosity coefficient of the blood, r is the radius of the tube, and r' is the mean radius of erythrocytes, which is estimated to be 3μ. The correction is not necessarily applied to viscosity coefficient, and it does not influence the result at all, what term in Hagen-Poiseuille's formula is adjusted. In the present investigation the length of arterial branches was adjusted instead of viscosity coefficient by:

$$l' = l \Big/ \left(1 + \frac{3 \times 10^{-4}}{r}\right)^2,$$

in which l is the anatomical length of arterial branches determined by measurement in cm, and l' is the effective branch length.

Now, it is possible to transform Hagen-Poiseuille's formula. The classical formula expressed in the form:

$$\Delta p = \frac{8\eta l Q}{\pi r^4},$$

in which Δp is pressure difference between the both ends of the tube in dyne/cm^2, η is the viscosity coefficient of the fluid in gm/cm. sec., l is the length of the tube in cm, Q is the flow in ml/sec. and r is the radius of the tube, is transformed on account of the relation $Q=qr^n$ to:

$$\Delta p = \frac{8\eta}{\pi} \cdot q \cdot \frac{l'}{r^{4-n}}. \tag{2}$$

If the length is expressed in μ, Δp in mm Hg, time in second, and if the viscosity coefficient of the blood is 0.035 gm/cm. sec. and n is 2.7, the formula (2) will be:

$$\Delta p = K \cdot q \cdot \frac{l'}{r^{1.3}} \tag{3}$$

and

$$l' = l \Big/ \left(1 + \frac{3}{r}\right)^2.$$

K is a common constant for all arterial systems and has the value:

$$K = 8 \times 0.035 \times 10^{12} \Big/ 3.1416 \times 980 \times 13.6 \times 10^{-1} = 6.686 \times 10^7.$$

Specific blood flow q is an organ specific constant and has a value in the order of 10^{-9} ml/sec. The last term $l'/r^{1.3}$ is determined by anatomical properties of

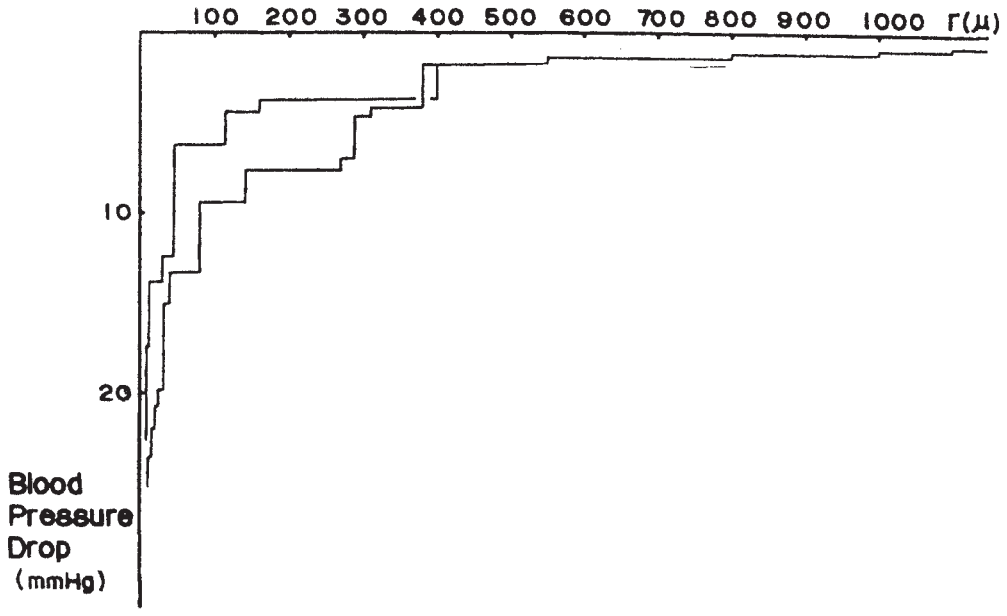

Fig. 8. Blood pressure drop calculated on arbitrarily selected routes of arterial casts of mesenteric artery.

arterial branches and can be given by actual measurements. Thus, formula (3) indicates that the blood pressure difference between the both ends of an unbranching arterial stretch is expressed by the product of an organ unspecific constant, an organ specific constant and a quantity determined by the anatomical property of arterial branches. The ratio $l'/r^{1.3}$ can be successively determined from the trunk to the arterioles of any arterial system by macroscopical and microscopical observation and measurement on the cast by way of any arbitrarily selected route. The total blood pressure difference is then given by $\Sigma \Delta p = \Delta P = K \cdot q \cdot \Sigma \frac{l'}{r^{1.3}}$, if $\Sigma \Delta p$ is designated as ΔP. In Fig. 8, examples obtained by this procedure from a mesenteric artery are demonstrated. The method is in this way adequate for hemodynamical analysis of individual routes of blood flow, but it is still inconvenient for theoretical consideration, because r is a discontinuous variable. Some devices are required to construct a model of arteries, in which r can be treated as a continuous variable.

CONSTRUCTION OF ARTERIAL MODEL FOR THEORETICAL TREATMENTS OF INTRAVASCULAR BLOOD PRESSURE

The transformed Hagen-Poiseuille's formula indicated that the influence of anatomical properties of arteries on blood flow could be summarized in the ratio $l'/r^{1.3}$. This suggests the importance of more detailed information about the relation of arterial radius to the effective branch length. An arterial branch maintains approximately its original radius until large subbranches are divided from it. But arterial branches are not strictly geometrical structures, and it is necessary to set a certain range of errors in the determination of their radii. In the present study the range of errors was taken as 5%. For example, an arterial branch of 200μ in radius was regarded to maintain its original radius, so far as the measurement gave values from 190μ to 210μ. Branches with irregular bulging and stricture were excluded from the investigation. It was further noticed that some arterial branches were slightly constricted just at their exits from larger branches and acquired constant size in distances nearly equal to their diameters. In such cases, the radius was determined on the stretch where they attained a stationary state. When in relatively rare instances arterial branches were extremely short, they did not usually attain a stationary state, and it was sometimes difficult to decide whether they can be regarded to be branches with definite length. In the present study, arterial branches shorter than their diameters were excluded from the measurement. Under the above conditions the radius and the length of randomly selected arterial branches were determined from the region of arterial trunk to arterioles. Because the length of a branch must be evaluated differently according to differently selected routes of blood stream, when only small subbranches divided in the course of the stretch, every possible route was taken into consideration. The procedure is diagrammatically explained in Fig.

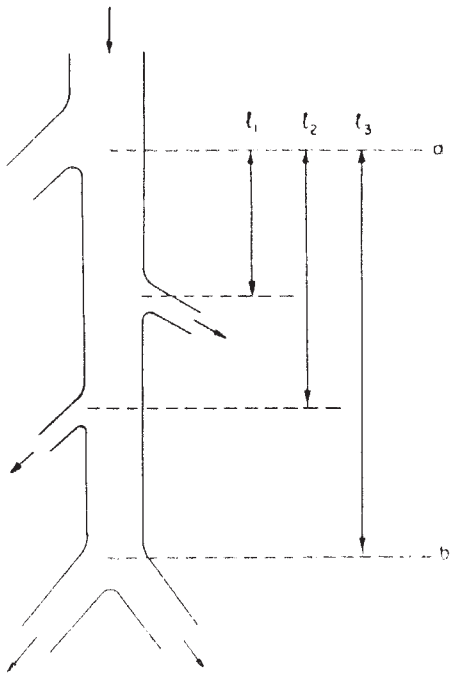

Fig. 9. We assume that an arterial branch has a radius r at a and $r \pm 0.05r$ along the stretch from a to b. Divisions of small subbranches do not cause noticeable reduction in radius. The length of the branch must have accordingly three different values l_1, l_2 and l_3 corresponding to three different possibilities in the selection of the route for blood stream. In the present investigation, l_1, l_2 and l_3 are all registered as the length of the branch.

9.

The results of measurements were plotted on a logarithmic scale taking the abscissa for the radius and the ordinate for the effective branch length. For a given r, the values of l' are highly scattered, the range of deviations being usually over 20 times. The result corresponds to the confirmed observation, that the length of arterial branches of equal radii can be extremely divergent. However, when a large number of measurements was plotted on the diagram, a distinct positive correlation was confirmable between r and l'. The relation appeared to be a linear regression on a logarithmic scale. On this assumption, the regression equations were determined and the relation of r and l' could be expressed by a general formula $l'=hr^i$, h and i being constants.

Table III shows the results of the determination on several arterial systems. The values of i are about 0.8 in renal artery, about 1.0 in femoral and mesenteric arteries and about 1.2 in cerebral artery. The lower the value of i is, the higher is the ratio of the effective length of arterioles to that of larger branches and the

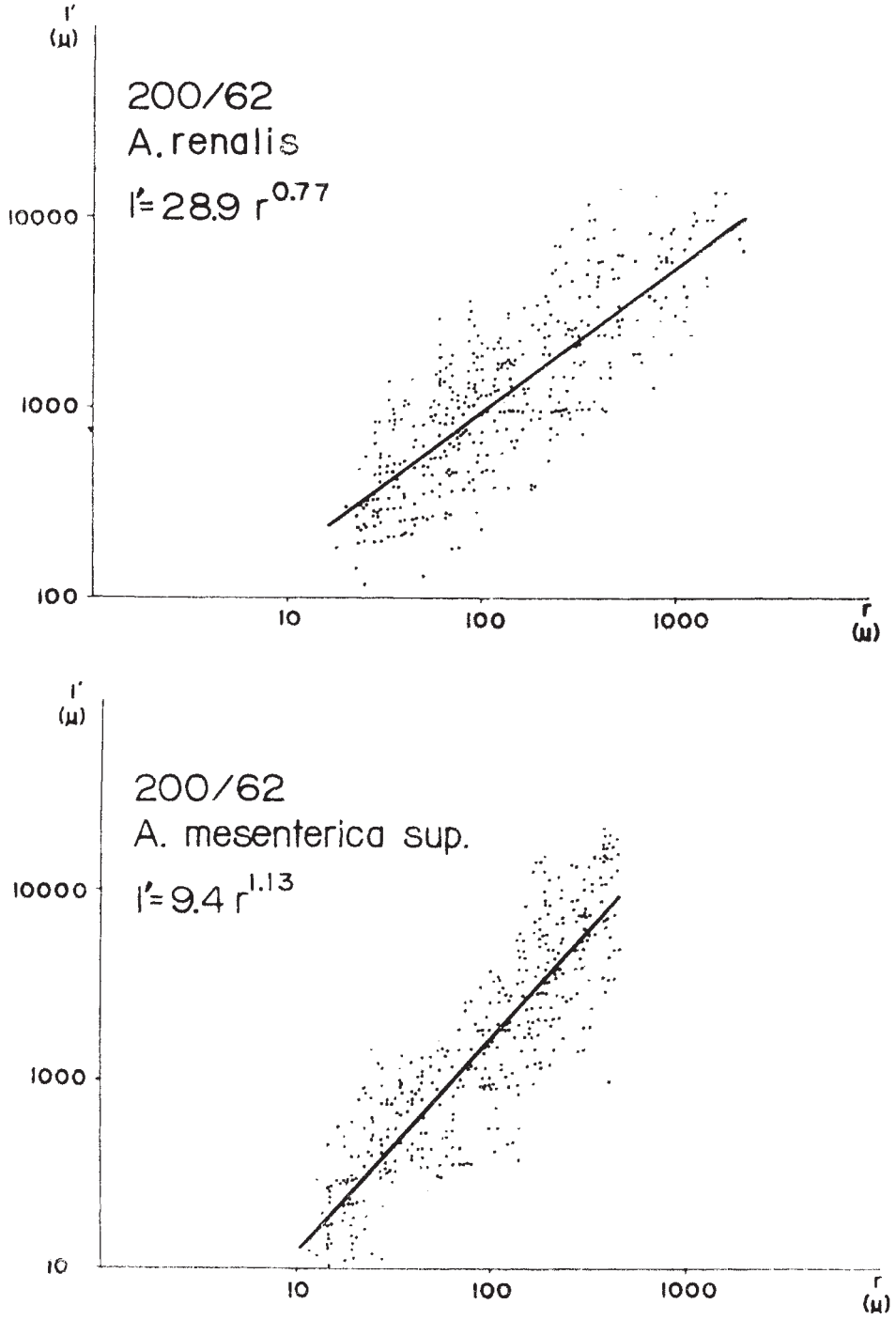

Fig. 10. The radius r and the effective length l' of arterial branches of renal and mesenteric arteries are demonstrated on the logarithmic scale. In spite of highly scattered individual values linear regression is confirmed between the two variables when a large number of measurements is plotted. Note the difference in the slope of regression equations of the two arterial systems. In mesenteric artery, the region $r>450\mu$ is not included in the determination of the regression equation. The region is characterized by abundant networks of arterial anastomosis, so that arterial branches are too short to be treated in a common sample group together with the region $r\leq 450\mu$. Accordingly, when an exact treatment is required, the total range of mesenteric artery must be divided into the two parts with different regression equations. However, because blood pressure drop in the region $r>450\mu$ is in any way only insignificant, the equation of the region $r\leq 450\mu$ is extrapolated to larger arterial branches. In the arterial systems examined in the present study, mesenteric artery was the only exception, in which anatomical structures made the separation of arterial ranges in reference to radius necessary.

TABLE III.

Organ or artery	Case	Regression equation	Degrees of freedom	Sx^2	$Sd^2y.x$	$l'=hr^i$
Renal artery	202/61	$\hat{Y}=0.8813X+1.1184$	379	72.7038	25.3011	$l'=13.1r^{0.88}$
	34/62	$\hat{Y}=0.8544X+1.2816$	404	87.7061	31.5274	$l'=19.1r^{0.85}$
	136/62	$\hat{Y}=0.8962X+1.1707$	519	155.9319	48.6836	$l'=14.8r^{0.89}$
	200/62	$\hat{Y}=0.7661X+1.4601$	433	102.7281	31.2063	$l'=28.8r^{0.76}$
	Common	$\hat{Y}=0.8530X+1.2463$	1738	419.0699	137.8385	$l'=17.6r^{0.85}$
Mesenteric artery	136/62	$\hat{Y}=1.0010X+1.2626$	418	84.1382	36.9877	$l'=18.3r^{1.00}$
	271/62	$\hat{Y}=0.9710X+1.1923$	491	56.4307	44.1493	$l'=15.5r^{0.97}$
	200/62	$\hat{Y}=1.1259X+0.9739$	434	90.3920	35.3162	$l'=9.4r^{1.12}$
	Common	$\hat{Y}=1.0425X+1.1170$	1345	230.9609	117.5131	$l'=13.0r^{1.04}$
Femoral artery	136/62	$\hat{Y}=0.9917X+1.2474$	348	145.7275	34.8327	$l'=17.6r^{0.99}$
	271/62	$\hat{Y}=1.0552X+0.9827$	426	100.7703	28.4637	$l'=9.6r^{1.05}$
	Common	$\hat{Y}=1.0177X+1.1228$	777	246.4978	63.5263	$l'=13.2r^{1.01}$
Pancreas	136/62	$\hat{Y}=0.9068X+1.2084$	285	66.2852	18.8254	$l'=16.1r^{0.90}$
Cerebral cortex	297/61	$\hat{Y}=1.1596X+0.8692$	497	75.8598	45.9948	$l'=7.4r^{1.15}$
Basal ganglion	297/61	$\hat{Y}=1.2178X+0.6660$	447	34.8958	31.5951	$l'=4.6r^{1.21}$
Heart	271/62	$\hat{Y}=1.0587X+0.9012$	425	106.0949	27.7134	$l'=7.9r^{1.05}$
Lung	102/62	$\hat{Y}=1.1601X+0.4520$	405	150.2339	39.0580	$l'=2.8r^{1.16}$

larger is the proportion of small branches in the total arterial length. If the effective length of arterial branches at $r=10\mu$ is calculated taking the effective length at $r=1000\mu$ as the unit length, it is about 1/40 for $i=0.8$ and 1/250 for $i=1.2$. The higher weight of arterioles in renal artery in comparison to other arteries is clearly demonstrated. The arterial pattern defined by $l'=hr^i$ has a decisive influence on the type of intravascular blood pressure gradient, which will be discussed later.

On the basis of the relation $l'=hr^i$, it is now possible to construct an arterial model adequate for hemodynamical analysis. For the sake of simplicity, successive dichotomy of arterial branches is postulated. If we take a certain route of blood stream out of the system, the radii of the branches constituting the route make a series r_x, $r_{x+1}, \ldots\ldots, r_{x+m-1}$, m being the total number of branches belonging to the route. The relation of the radius of a branch r_{x+j} to that of the next subbranch r_{x+j+1} is defined by $r_{x+j+1}=\delta r_{x+j}$, and it is further assumed that δ has the same value throughout the series. The total effective length of the route λ_x' from the branch with r_x downward is given by:

$$\lambda_x' = l_x' + l_{x+1}' + \ldots\ldots + l_{x+m-1}', \qquad (4)$$

if l_x', $l_{x+1}', \ldots\ldots, l_{x+m-1}'$ represent the effective branch length, respectively. On account of the relation $l'=hr^i$, the formula (4) will be:

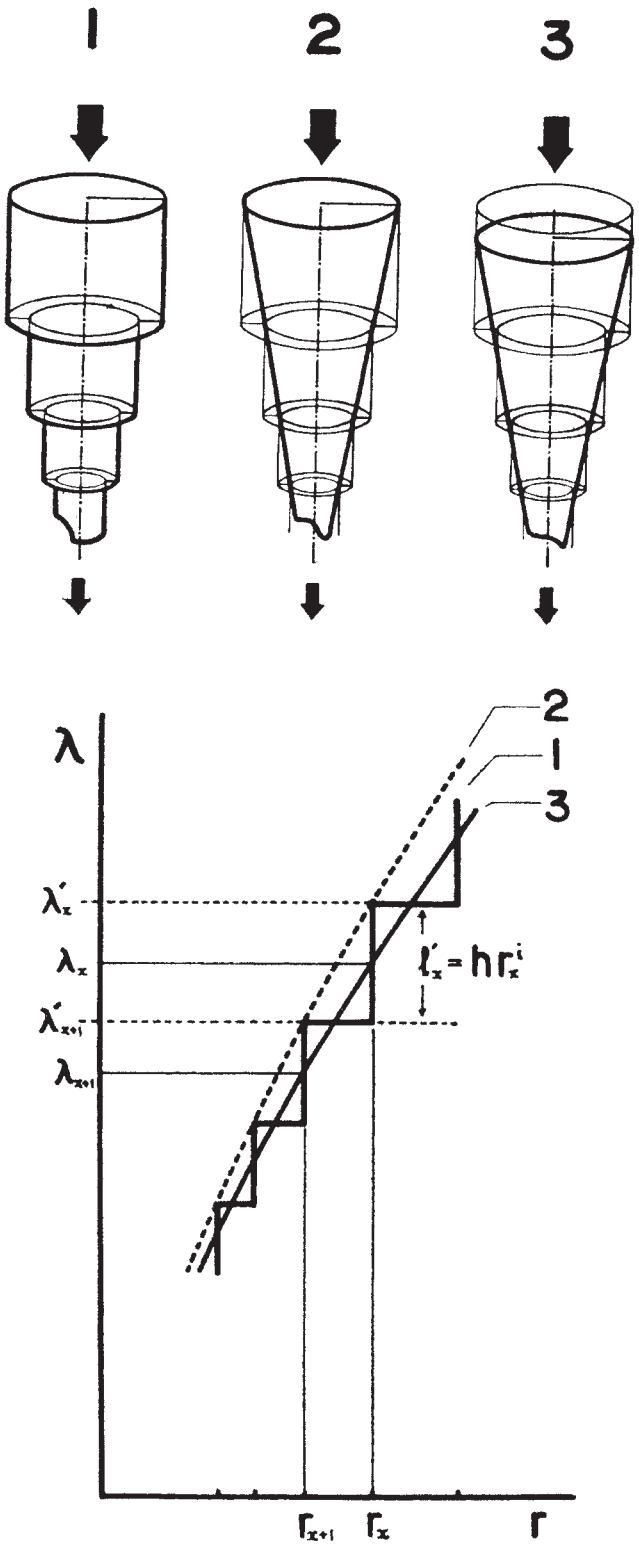

Fig. 11. Arterial models are presented with the lines of their walls on the plane which cuts the models including their tube axes. According to $i>1$, $i=1$ and $i<1$, the wall of the models with continuous r is concave upwards, straight and convex upwards, respectively.

$$\lambda_x' = hr_x^i + hr_{x+1}^i + \cdots\cdots + hr_{x+m-1}^i$$
$$= hr_x^i(1 + \delta^i + \delta^{2i} + \cdots\cdots + \delta^{(m-1)i}).$$

If we assume infinite subdivision of arterial branches,

$$1 + \delta^i + \delta^{2i} + \cdots\cdots + \delta^{(m-1)i}$$

is the sum of a convergent infinite geometrical series, because i and δ are positive and δ is smaller than 1. Accordingly, we obtain:

$$\lambda_x' = \frac{h}{1-\delta^i} \cdot r_x^i. \tag{5}$$

The formula (5) corresponds to Model 1 in Fig. 11. The actual shape of the model is a straight tube with circular cross section, which reduces its radius discontinuously at the end of each step. If the tube is cut by a longitudinal plane passing the tube axis, the wall of the tube is represented by the step-like line in Fig. 11. It is assumed that blood flow in the tube is constant as long as the radius is constant and reduces its quantity abruptly at the distal end of each step, so that blood flow is always adjusted to qr^n. Blood pressure drop in the stretch from r_x to r_{x+1} of the tube is:

$$\Delta P_1 = K \cdot q \cdot \frac{l_x'}{r_x^{4-n}} = K \cdot q \cdot hr_x^{n+i-4} = K \cdot q \cdot hr_x^g \tag{6}$$

if $n+i-4$ is designated as g. As formula (6) is a power function of r, we can treat it as a continuous function of r and express it as:

$$\lambda' = \frac{h}{1-\delta^i} \cdot r^i. \tag{7}$$

The equation (7) corresponds to Model 2 in Fig. 11. The shape of the second arterial model is now a continuously tapering straight tube, which inscribes Model 1 and through the wall of which continuous blood leakage takes place, so that the blood flow in the tube is always equal to qr^n. Blood pressure drop from r_x to r_{x+1} in this arterial model is:

$$\Delta P_2 = K \cdot q \cdot \int_{\lambda x+1}^{\lambda x} \frac{d\lambda'}{r^{4-n}} = K \cdot q \cdot \frac{h}{1-\delta^i} \int_{r_{x+1}}^{r_x} r^{g-1} \cdot dr$$

$$= K \cdot q \cdot \frac{ih}{g(1-\delta^i)} \cdot (r_x^g - r_{x+1}^g)$$

$$= K \cdot q \cdot hr_x^g \cdot i(1-\delta^g)/g(1-\delta^i). \tag{8}$$

Except in the case of $g=i$, or $n=4$ as $g=n+i-4$, the ratio $i(1-\delta^g)/g(1-\delta^i)$ is not equal to 1, and $\Delta P_1 \neq \Delta P_2$. A third arterial model, Model 3, must be therefore introduced, in which blood pressure drop between two given radii is exactly equal to that of Model 1. If the equation of the required model is written as $\lambda = Z\lambda'$,

the value of Z is determined by $\Delta P_1/\Delta P_2$ and we obtain $Z=g(1-\delta^i)/i(1-\delta^g)$. The equation for the required model is:

$$\lambda = \frac{gh}{i(1-\delta^g)} \cdot r^i . \tag{9}$$

The equation (9) perfectly satisfies the requirement for the theoretical determination of intravascular blood pressure gradient. The radius can be treated as a continuous variable and blood pressure drop between two given radii is exactly the same to that in the first model with discontinuous r. The blood pressure drop between r_x and r_{x+1} of Model 3 is:

$$\Delta P_3 = K\cdot q\cdot \int_{\lambda_{x+1}}^{\lambda_x} \frac{d\lambda}{r^{4-n}} = K\cdot q\cdot \frac{gh}{1-\delta^g} \int_{r_{x+1}}^{r_x} r^{g-1}\cdot dr$$

$$= K\cdot q\cdot \frac{h}{1-\delta^g} \cdot (r_x{}^g - r_{x+1}{}^g) = K\cdot q\cdot h r_x{}^g = \Delta P_1 .$$

If the indefinite integral $hr^g/(1-\delta^g)+C$ is designated as τ, blood pressure drop between any two given radii r_1 and r_2 $(r_1>r_2)$ is given by:

$$\Delta P = K\cdot q\cdot \frac{h}{1-\delta^g} \cdot \left[r^g\right]_{r_2}^{r_1} = K\cdot q\cdot \left[\tau\right]_{r_2}^{r_1}, \tag{10}$$

which represents the substitution of $\Sigma \frac{l'}{r^{4-n}}$ by τ. Further it is possible with (9) to calculate the mean effective length of an arterial system between two given radii by:

$$\lambda_{r_2}^{r_1} = \frac{gh}{i(1-\delta^g)} \cdot (r_1{}^i - r_2{}^i) .$$

When $g=0$, the equation (9) is indefinite. In this case, the numerator and denominator of the equation are differentiated in reference to g, respectively, and the equation can be written as:

$$\lambda = \frac{h}{i(-\log \delta)} \cdot r^i .$$

The blood pressure drop from r_1 to r_2 $(r_1>r_2)$ is given in the case of $g=0$ by:

$$\Delta P = \frac{K\cdot q\cdot h}{-\log \delta} \left[\log r\right]_{r_2}^{r_1} .$$

The influence of varying g on intravascular blood pressure gradient is demonstrated in Fig. 12. When g is negative, intravascular blood pressure drop is represented by a curve convex upwards on a semilogarithmic scale. This means that blood pressure drop between $r=1000\mu$ to $r=100\mu$ is smaller than that between $r=100\mu$

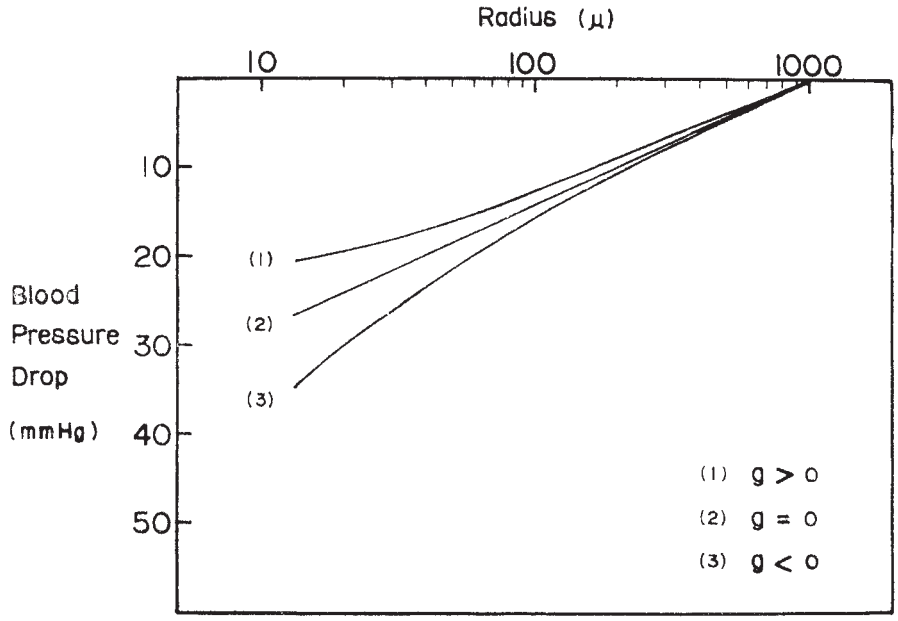

Fig. 12.

and $r=10\mu$. There is an acceleration of blood pressure drop on a semilogarithmic scale in the region of small arteries. The majority of examined arterial systems has negative g and is characterized by blood pressure gradient of this pattern. When g is positive, the curve is concave upwards, and blood pressure drop from $r=1000\mu$ to $r=100\mu$ is larger than that from $r=100\mu$ to $r=10\mu$, which indicates deceleration of blood pressure drop in small arteries. When $g=0$, intravascular blood pressure drop is represented by a straight line, and blood pressure drop from $r=1000\mu$ to $r=100\mu$ is exactly equal to that from $r=100\mu$ to $r=10\mu$. In actual arterial systems, the latter two conditions can occasionally be attained especially by arteries for basal ganglia of the brain.

We have not yet examined, how to treat δ in the construction of arterial models. We assume now, that an arterial branch sends off subbranches by infinite successive dichotomy. We further assume that the relation of the radius of each of these branches r to those of the next subbranches, r_a and r_b, are always defined by $r_a=\delta r$ and $r_b=\varepsilon r$. Out of the system, we take an extreme route, the radii of which make a series r, δr, $\delta^2 r$, ..., $\delta^m r$ $(m\to\infty)$. The other extreme route has then the series of radii, r, εr, $\varepsilon^2 r$,..., $\varepsilon^m r$ $(m\to\infty)$. For the first route, blood pressure drop between two arbitrarily selected radii r_1 and r_2 $(r_1>r_2)$ is given by:

$$\Delta P_a = K\cdot q\cdot \frac{h}{1-\delta^g}\cdot \left[r^g\right]_{r_2}^{r_1}$$

and for the second route,

$$\Delta P_b = K\cdot q\cdot \frac{h}{1-\varepsilon^g}\cdot \left[r^g\right]_{r_2}^{r_1}.$$

Because the blood pressure drop between r_1 and r_2 of all the other possible routes taken out of the system has some values between ΔP_a and ΔP_b, we may represent the blood pressure drop of the total system for example by:

$$\Delta P = (\Delta P_a + \Delta P_b)/2$$

$$= \frac{1}{2}\left(\frac{1}{1-\delta^g} + \frac{1}{1-\varepsilon^g}\right) \cdot K \cdot q \cdot h \left[r^g\right]_{r_2}^{r_1} = \theta \cdot K \cdot q \cdot h \left[r^g\right]_{r_2}^{r_1},$$

if the term $\frac{1}{2}\left(\frac{1}{1-\delta^g} + \frac{1}{1-\varepsilon^g}\right)$ is designated as θ. Our purpose is to examine the variation of θ according to the variations of δ and ε. Because of the relation $\delta^n+\varepsilon^n=1$, as $r_a{}^n+r_b{}^n=r^n$, the differential $d\theta/d\delta$ is given by:

$$\frac{d\theta}{d\delta} = \frac{g}{2}\left[\frac{\delta^{g-1}}{(1-\delta^g)^2} - \left(\frac{\delta}{\varepsilon}\right)^{n-1} \cdot \frac{\varepsilon^{g-1}}{(1-\varepsilon^g)^2}\right]. \tag{11}$$

If the ratio of the second to the first term in the large parenthesis is designated as α, we obtain:

$$\alpha = \left(\frac{\delta}{\varepsilon}\right)^{n-1} \cdot \frac{\varepsilon^{g-1}}{(1-\varepsilon^g)^2} \Big/ \frac{\delta^{g-1}}{(1-\delta^g)^2} = \frac{\delta^{n-g}(1-\delta^g)^2}{\varepsilon^{n-g}(1-\varepsilon^g)^2},$$

and the formula (11) is transformed to:

$$\frac{d\theta}{d\delta} = \frac{g}{2} \cdot \frac{\delta^{g-1}}{(1-\delta^g)^2} \cdot (1-\alpha).$$

Corresponding to $\delta>\varepsilon$, $\delta=\varepsilon$ and $\delta<\varepsilon$, the value of α is $\alpha>1$, $\alpha=1$ and $\alpha<1$, respectively, if g is negative. Accordingly, $d\theta/d\delta>0$, $d\theta/d\delta=0$ and $d\theta/d\delta<0$ corresponding to $\delta>\varepsilon$, $\delta=\varepsilon$ and $\delta<\varepsilon$, respectively. This demonstrates that θ takes its minimum value when $\delta=\varepsilon$. The results indicates that out of possible models for dichotomic arterial systems the minimum blood pressure drop is obtained by the one which postulates successive subdivision into branches of equal size. In this case, δ is given by $\delta=2^{-1/n}$, and when n is 2.7, we obtain 0.7736 for δ.

If successive subdivision into more than two equal branches is assumed, δ is determined by $\delta=m^{-1/n}$, m being the number of subbranches at a single division. For trichotomy the value is 0.6657, if n is 2.7. Therefore, the absolute value of $1/(1-\delta^g)$ is smaller in trichotomy than in dichotomy, and blood pressure drop between two given radii is smaller in the case of trichotomy than in dichotomy. In the actual arterial systems examined, the majority of arterial divisions is clearly represented by dichotomy, and other modes of division can be regarded to be more or less exceptional. Only in organs with very short arterial branches such as lung and kidney, divisions other than dichotomy are comparatively frequently observed. Accordingly, divisions other than dichotomy are

neglected in the following sections of this report. In more accurate investigations of pulmonary and renal arteries, however, some corrections will be necessary.

DETERMINATION OF INTRAVASCULAR BLOOD PRESSURE GRADIENT BY MEANS OF THE ARTERIAL MODEL

With the determination of δ, all necessary values for the calculation of intravascular blood pressure by formula (10) were obtained. Strictly speaking, our method is only applicable on the organs with known blood flow quantity. However, as the values of q were not greatly different among the examined organs of known blood flow, q was fixed to 5.0×10^{-9} ml/sec. regardless of organ difference, so far as the comparison of the influence of anatomical property of arterial system was concerned, and the application of the method was extended to the organs of which there was no available information of the blood flow.

In the calculation of ΔP with (10), it is practically not necessary to calculate the term $\left[r^g\right]_{r_2}^{r_1}$. When the absolute value for $K\cdot q\cdot h/(1-\delta^g)$ is determined, it is plotted on the ordinate of a logarithmic scale at $r=1\mu$, and through this point a straight line is drawn with the slope of $\tan\theta=g$. On this line, the ordinate corresponding to any two arbitrarily selected radii can be determined. The difference between the two readings gives the value of blood pressure drop between the two radii with accuracy sufficient for our purpose. An example with the common regression equation of mesenteric artery is demonstrated in Fig. 13.

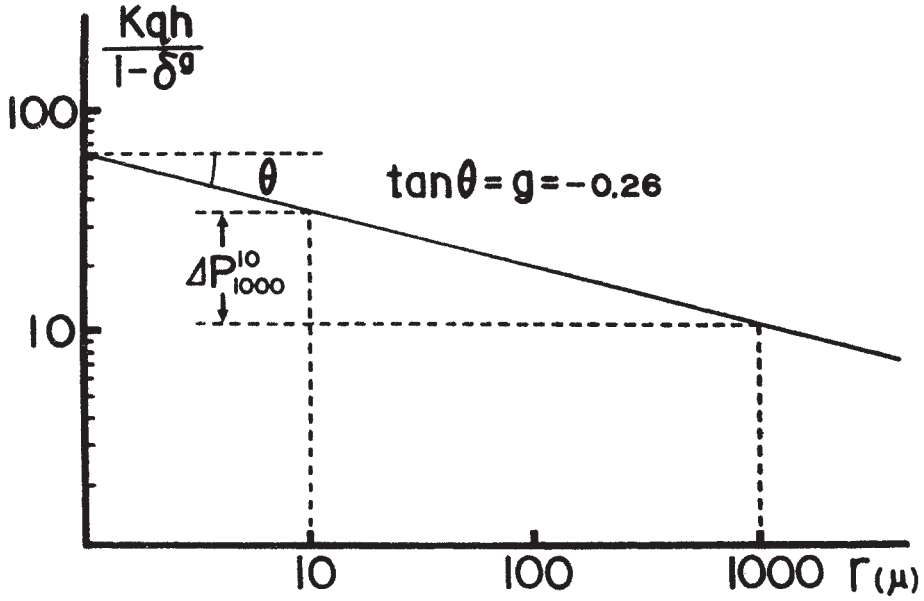

Fig. 13. An example for the determination of blood pressure drop on a logarithmic scale. As $K\cdot q\cdot h/(1-\delta^g)$ is negative, its absolute value is taken on the scale. The ordinate on the line with the slope g corresponding to a given radius on the abscissa represents blood pressure drop from $r=\infty$ to the given radius.

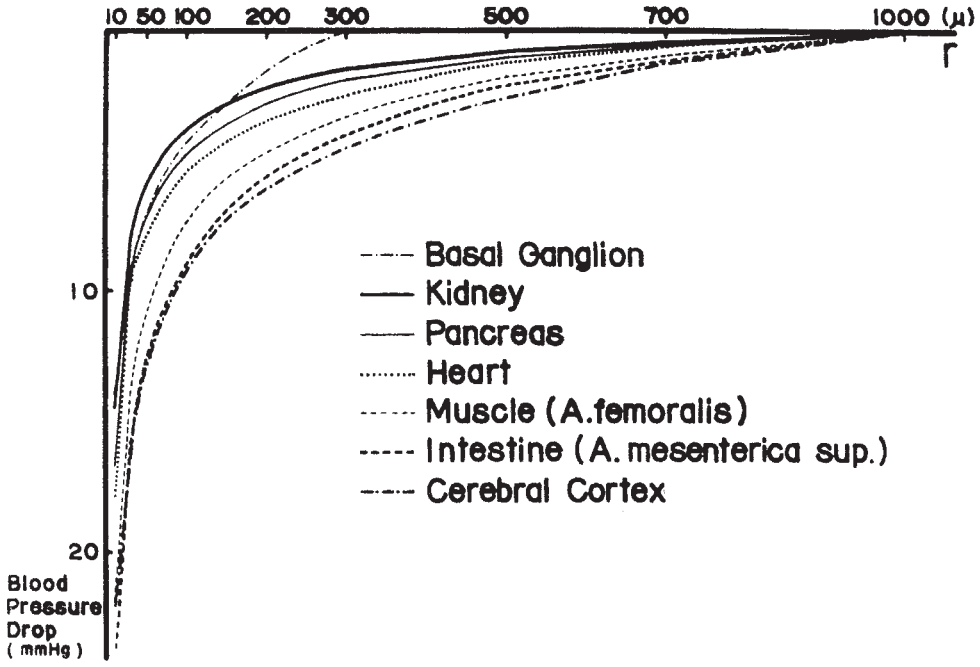

Fig. 14.

The results of blood pressure estimation on several arterial systems are diagrammatically presented in Fig. 14. In every arterial system remarkable blood pressure drop takes place only in the region of arteries smaller than 100μ in radius. Our results correspond in this respect to the works reported by previous investigators. At the same time, some differences are noticed in the pattern of blood pressure gradient due to the difference of anatomical property of arteries according to organs. In renal artery, blood pressure is only insignificantly lowered in the region of larger arterial branches and falls abruptly in arterioles. On the contrary, the patterns of cerebral, mesenteric and femoral arteries are characterized by relatively distinct pressure drop in arterial branches of larger radius and by comparatively mild acceleration in blood pressure drop in the arteriolar region. The difference in the pattern of blood pressure gradient is due to the different values of i in $l'=hr^i$. The larger the value of i is, the greater is the weight of larger branches in the total resistance to blood flow in the arterial system.

Another important difference is noticed in the calculated blood pressure level in the arteriolar region. When blood pressure level is checked at $r=10\mu$, it is distinctly lower in mesenteric, femoral and cerebral arteries than in renal and pancreatic arteries. This indicates that the arteriolar region in the latter organ group is liable to be exposed to higher blood pressure than that in the former, so far as normal blood flow is sustained and arterial branches are dilated. The major anatomical factor which causes the difference is the effective length of arterial systems. In Fig. 15, the effective length of examined arterial systems from $r=$

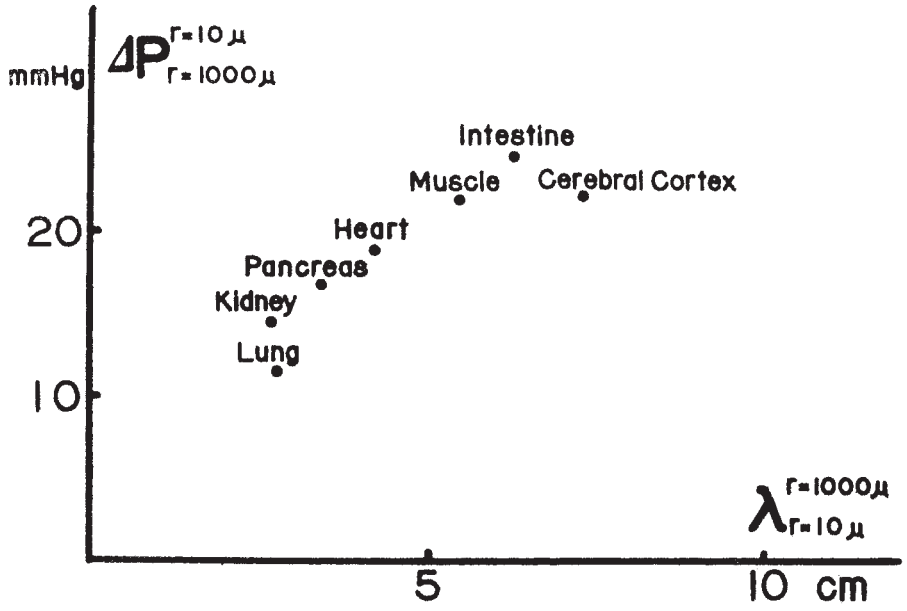

Fig. 15.

1000μ to $r=10\mu$ is presented in reference to the blood pressure drop in the same region. It is clearly demonstrated that the value of λ is distinctly higher in the arterial group with large blood pressure drop. It appears that arterial system is unable to adjust hemodynamical difference due to different anatomical length of arteries completely, on account of relatively restricted possibility in the variation of other anatomical factors which can influence blood flow. The value of n is almost constant, dichotomy is dominating in every arterial system of large effective length and the only possible compensation of arterial length by the difference of i seems to be still insufficient to equalize blood pressure level in the arteriolar region. If the actual blood pressure level in the arteriolar region is practically the same in every part of the living organism, it is in all probability only achieved by different muscular tension of arterial walls.

It is further interesting that arterial systems with high estimated blood pressure level in the region from 100μ to 200μ in radius belong to the arterial group which is associated with high incidence of hypertensive arteriolar lesions. Except for arteries of basal ganglion of the brain, which divide immediately from large arteries already as branches smaller than 300μ in radius, this group is characterized by a low value of i. Such arteries are characterized by only insignificant blood pressure drop in their larger branches and by extremely steep blood pressure gradient in their arteriolar region. Although it is impossible to discuss the mechanism of hypertensive circulatory disturbance in this report, our results suggest the importance of hemodynamical arterial pattern as one of the predisposing factors to hypertensive arteriolar injuries.

ESTIMATION OF ARTERIAL RADIUS IN LIVING ORGANISM

In previous sections of this report, determinations of intravascular blood pressure gradient were entirely based on the results obtained directly from arterial casts, without examinations of possible discrepancies between arterial casts and living arteries. In this section of the report attempts will be made to estimate the state of living arteries by adjusting the values of blood pressure drop calculated on our arterial model.

Arterial casts represent more or less distended arterial lumina. This is not only expected from the manipulation of resin infusion under unphysiologically high pressure, but also concluded from the values of blood pressure drop. According to Landis,[5] the blood pressure level of arterioles of human skin is about 30 mm Hg. If blood pressure at the terminal part of mesenteric artery is likewise of the same level and the mean blood pressure of the trunk of mesenteric artery is 90 mm Hg, the pressure drop from the trunk to arterioles must be about 60 mm Hg. The blood pressure drop from $r=1000\mu$ to $r=10\mu$ of mesenteric artery calculated on the model is about 25 mm Hg, which is about 42% of the assumed intravital blood pressure drop. If the artificial dilatation due to resin infusion were uniform in the whole range of arterial branches and blood flow were sustained at the same level, blood pressure drop would be inversely proportional to the fourth power of arterial radius. If we designate the radius of living artery corresponding to r of arterial cast as r_0, the ratio r/r_0 is then determined by $(r/r_0)^4=1/0.42$ or $r=1.25r_0$. The result indicates that artificial dilatation due to resin infusion would not exceed 25%.

However, it is to be examined whether artificial arterial distention is uniform in the entire arterial length or it is influenced by the size of arterial branch. The relation was examined by the following procedure.

In an autopsy case with normal kidneys acrylic resin was infused in one of the kidneys and the organ was fixed in formalin after infused resin had been solidified. The other kidney was fixed in formalin without resin infusion. Tissue slices were excised from both kidneys and embedded in paraffin. Solidified resin in arteries was completely dissolved and removed by organic solvents in the course of paraffin embedding. Histological examination revealed remarkable dilatation of arteries of the resin infused kidney and the dilatation was especially pronounced in small arterial branches. Histological arterial specimens were treated by the histometrical method described by Furuyama.[2] The essentials of the method were to reduce the arteries of autopsy cases to the state in which internal elastic membrane was perfectly stretched and to eliminate the effect of post-mortem arterial constriction. The radius (in this method the distance from the center of arterial lumen to the middle point of the media) was defined as anatomical arterial radius and designated as R. About 20 samples of exact arterial cross

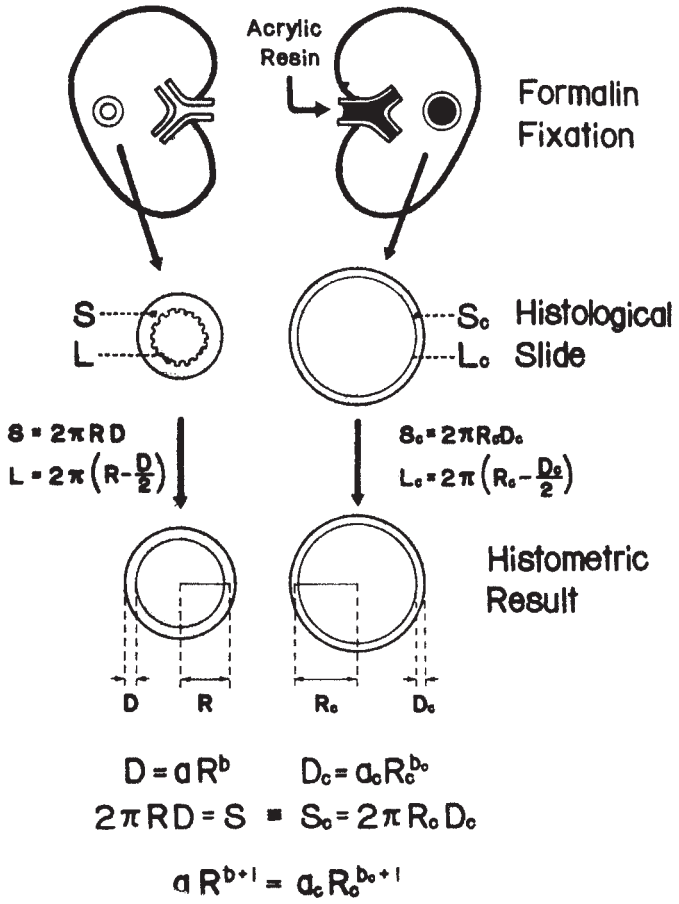

Fig. 16. The diagram of the method to determine the grade of artificial arterial dilatation due to resin infusion in comparison to the state of arteries with stretched internal elastic membrane without resin infusion. The principle of the method is to treat the surface area S of the media of arterial cross sections as an invariable quantity independent of the state of arterial constriction and to regard arterial branches with equal S as having equal radii.

sections of various size were selected from each kidney, and the length of the cross section of internal elastic membrane L and the surface area of the media S were determined on the magnified depictions of arterial cross sections. From L and S anatomical radius R and the medial thickness D corresponding to the state were calculated by:

$$R = \frac{S}{\sqrt{L^2 + 4\pi S} - L}$$

and

$$D = \frac{\sqrt{L^2 + 4\pi S} - L}{2\pi}$$

The symbol for the resin infused organ is indicated by c as in R_c and D_c. The

relation of R_c and D_c, as well as that of R and D, is a linear regression on a logarithmic scale, so that the general relation of R_c and D_c or R and D is given by general formulae $D_c=a_cR_c^{b_c}$ and $D=aR^b$, a, a_c, b and b_c being constants. The surface area of the media S on the arterial cross section can be regarded to be a quantity not influenced by the grade of arterial constriction. Arterial samples from the same organ of the same autopsy case with equal S are accordingly considered to have had equal radii in the original or living state. As $S=2\pi RD$, we obtain $RD=R_cD_c$, or, substituting D and D_c by aR^b and $a_cR_c^{b_c}$, respectively, $aR^{b+1}=a_cR_c^{b_c+1}$ as the equation which defines the relation between anatomical radius of the resin non-infused artery and the corresponding radius of the resin infused artery. For the renal artery of a normal adult autopsy case, we obtained by the above method:

$$R_c = 2.35\, R^{0.90}\,.$$

We repeated the examination likewise on other arterial systems and obtained essentially the same results. This indicates that the measurements of arterial casts give an about two times larger value as the estimate of arterial radius than the anatomical radius at $R=10\mu$, but the difference is reduced to an almost negligible degree at arterial trunk. The result demonstrates that artificial arterial dilatation due to resin infusion is not uniform in the total range of arterial branches, but it is expressed by a power function of the anatomical radius. It is not revealed, however, whether the anatomical radius is identical with the radius of living artery or not, but we assume that the relation between the radius of arterial cast and that of living artery can be likewise expressed in the form of a power function, $r=ur_0^w$, in which r and r_0 are the radius of arterial cast and living artery, respectively, and u and w are constants. Now we can determine the values of u and w as follows. We take for example mesenteric artery and assume that $r=r_0=3000\mu$ at the trunk of mesenteric artery and $r=1.25r_0$ at $r_0=10\mu$. On these assumptions the following equations are determined:

$$\begin{cases} 3000 = u \cdot 3000^w \\ 12.5 = u \cdot 10^w\,. \end{cases}$$

From these equations we obtain $u=1.369$ and $w=0.9609$. The correction equation under the above conditions is accordingly given by:

$$r = 1.369\, r_0^{0.9609}\,.$$

When the correction equation is applied to our arterial model, the values of h, i, q, n, g and δ are necessarily adjusted, and we obtain a new arterial model with different constants from those of the model based on the direct measurements of arterial casts. The calculation of blood pressure drop with the adjusted model is completely the same as was explained in previous sections of this report. To the

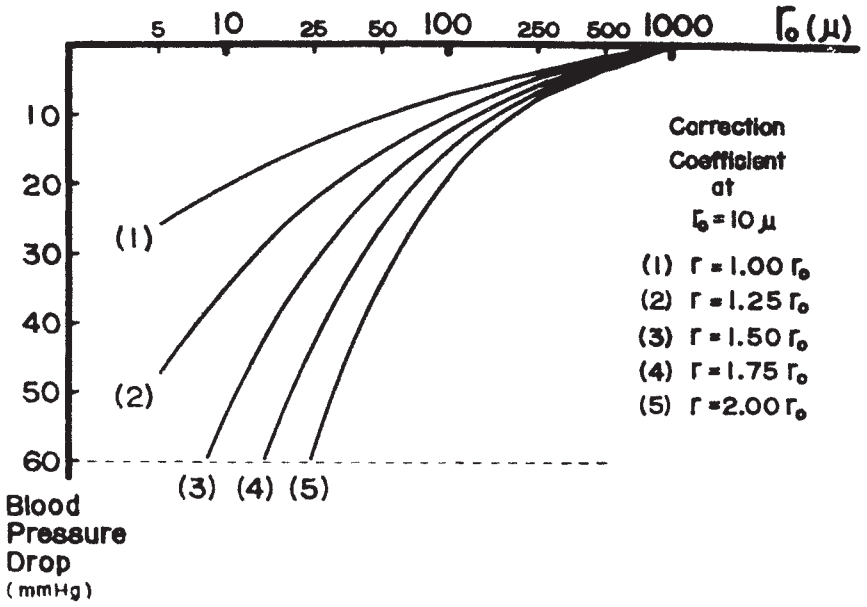

Fig. 17.

TABLE IV.

Coefficient of dilatation at $r_0=10\mu$	Correction equation $r=ur_0^w$	h	i	n	q (10^{-9})	g	δ	ΔP_{10}^{1000} (mmHg)
1.00	$r=1.000r_0^{1.0000}$	13.00	1.0425	2.7000	4.20	−0.2600	0.7736	20.6
1.25	$r=1.369r_0^{0.9609}$	18.15	1.0018	2.5944	9.74	0.4038	0.7655	35.1
1.50	$r=1.767r_0^{0.9289}$	23.70	0.9685	2.5081	19.45	−0.5235	0.7585	53.7
1.75	$r=2.194r_0^{0.9019}$	29.70	0.9403	2.4351	34.89	−0.6247	0.7523	79.3
2.00	$r=2.646r_0^{0.8785}$	37.88	0.9159	2.3720	57.76	−0.7121	0.7466	116.5

The figures in the table were calculated on the assumption that blood flow of mesenteric artery was 10.8 ml/sec. in every arterial model.

coefficient of dilatation at $r_0=10\mu$, we can give varying values and we obtain a series of adjusted arterial models. In Fig. 17 and Table IV the values of blood pressure drop calculated with several adjusted models are presented together with the values of pertaining constants. When the coefficient of dilatation at $r_0=10\mu$ is set over 1.75, calculated blood pressure drop from $r_0=1000\mu$ to $r_0=10\mu$ is evidently too large and exceeds 60 mm Hg. In other words, the radius of arterial casts can not be larger than 1.75 times of that of living artery at 10μ in radius. From the diagram it may be concluded, that artificial arterial dilatation due to resin infusion is most probably not much different from 50% of living artery at $r_0=10\mu$. In the larger arterial branches resin infusion will cause less remarkable dilatation.

By means of the above procedure, we could adjust our arterial models to the form which might be regarded to correspond to intravital arterial state more

closely. However, the results of the correction did not change the conclusions obtained in previous sections of this report, so far as the comparative study of intravascular blood pressure gradient of different arterial systems was concerned. We think it rather appropriate to omit an extensive description of the results with adjusted arterial models in this report.

SUMMARY

1) Arterial blood flow was expressed as a power function of arterial radius. In the relation defined by $Q=qr^n$, Q was blood flow, q an organ specific constant, and n had a value about 2.7 regardless of organ difference.

2) On account of the above relation, Hagen-Poiseuille's formula was transformed to $\Delta p=K\cdot q\cdot l'/r^{4-n}$, in which K was an organ unspecific constant determined by viscosity coefficient of blood and selected units, l' was the effective length and r was the radius of an arterial branch. In the transformed formula, pressure difference between both ends of an unbranching stretch of arteries Δp was given by the product of organ unspecific and organ specific constants and a quantity determined by anatomical properties of arteries. By means of successive determinations of l'/r^{4-n} by way of any arbitrarily selected route, the total blood pressure drop could be determined by $\Sigma\Delta p=K\cdot q\cdot\Sigma\dfrac{l'}{r^{4-n}}$.

3) The relation of the effective branch length l' and the radius r of an arterial system was expressed by $l'=hr^i$, in which h and i were organ specific constants. A higher value of i indicated a higher proportion of large arterial branches in the total arterial length.

4) Assuming successive dichotomy of arterial branches, an arterial model was constructed with radius as a continuous variable. Blood pressure drop between any two given radii could be estimated by the calculation on the model by:

$$\Delta P=\frac{K\cdot q\cdot h}{1-\delta^g}\left[r^g\right]_{r_2}^{r_1}\ \text{for}\ g\neq 0\ \text{and}\ \Delta P=\frac{K\cdot q\cdot h}{-\log\delta}\left[\log r\right]_{r_2}^{r_1}\ \text{for}\ g=0\,.$$

5) Some organ differences in the pattern of blood pressure gradient and intravascular blood pressure values were confirmed and were discussed in reference to different anatomical properties of arterial systems. Renal artery was characterized by pronounced acceleration of blood pressure drop in the arteriolar region and by only insignificant pressure drop in its large branches. On the contrary, considerable blood pressure drop took place in large arterial branches of mesenteric, femoral and cerebral arteries in contrast to comparatively mild pressure drop in the arteriolar region. The estimated blood pressure level in the arteriolar region was correlated to the effective length of arterial systems. It was found to be higher in the arterial group with short effective arterial length. It was suggested that arterial systems with high susceptibility to hypertensive

arteriolar injuries belonged to arteries with high bood pressure level in the region of $r=100\mu$ to $r=200\mu$.

6) Artificial arterial dilatation due to resin infusion could be defined by a power function of arterial radius. The function could be used as the correction equation of the radius of arterial casts, and the radius of living arteries was estimated on the basis of blood pressure estimation with corrected arterial models.

Acknowledgment

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